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**of**

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**for**

**SYSTEMS AND METHODS FOR DITHER STRUCTURE  
CREATION AND APPLICATION**

## BACKGROUND

Digital images are communicated by values that represent the luminance and chromatic attributes of an image at an array of locations throughout the image. Each value is represented by a given number of bits. When bandwidth, storage and display requirements are not restrictive, sufficient bits are available that the image can be displayed with virtually uninhibited visual clarity and realistic color reproduction. However, when bit-depth is restricted, the gradations between adjacent luminance or color levels can become perceptible and even annoying to a human observer. This effect is apparent in contouring artifacts visible in images with low bit-depth. Contour lines appear in low frequency areas with slowly varying luminance where pixel values are forced to one side or the other of a coarse gradation step.

These contouring artifacts can be “broken up” by adding noise or other dither patterns to the image, generally before quantization or other bit-depth reduction. This noise or pattern addition forces a random, pseudo-random or other variation in pixel values that reduces the occurrence and visibility of contours. Typically, the image is perceived as more natural and pleasing to a human observer.

Some of these methods can be explained with reference to Figure 1, which illustrates an image display system 1. In these systems, noise or dither patterns 16 can be added to 4 or otherwise combined with an image 2. The combined image is then quantized 6 to a lower bit-depth. The image may then be displayed directly or , as shown in Figure 1, may be transmitted 8 to a receiver 10. After reception, the noise/dither 16 that was added to the image may be subtracted 12 or otherwise de-combined with the image to reduce the visible effect of the noise/dither on areas where

contouring is not likely to occur. The image is then displayed 14 on the receiving end. These methods may also be used in systems that do not transmit or receive such as with displays with bit-depth capabilities that are lower than the image data 2 to be displayed.

Some of these methods may be explained with reference to Figure 2. In these  
5 systems 20, an image 2 is combined 28 with a noise/dither pattern 16 and sent to a display system 22 that cannot display the full range of image data contained in the image. These display systems 22 may quantize 24 the image data to a bit-depth that matches the display capabilities. The quantized image data is then displayed on the display 26.

10 In the systems illustrated in Figure 2, the noise/dither pattern is not subtracted or de-combined from the image. In these systems, less noise can be added to an image before it causes adverse visual impact or “graininess.” Various frequency distributions for noise/dither patterns have been found to be more or less visible to the human visual system. Generally, the human visual system works as a low-pass filter that filters out  
15 high frequency data. Therefore, noise concentrated in a high-frequency range is less visible than lower frequency noise.

Often it is not feasible to use a dither/noise pattern that is as big as an image file. In these cases, a smaller dither pattern can be used by repeating the pattern across the image in rows and columns. This process is often referred to as tiling. In multiple  
20 image sets, such as the frames or fields of video images, a dither pattern may be repeated from frame to frame as well. Dither patterns may be designed to minimize artifacts created by their repetitive patterns.

Dither structures may comprise multiple dither patterns to be used across a

single image of multiple frames. A three-dimensional dither structure, as shown in Figure 3, may employ a series of dither patterns. These patterns 30–36 may be arranged in a sequence that is used on sequential frames of video. A first dither pattern tile 30 may be used on a first video frame 38 while a next sequential pattern 32 is used on a next successive video frame 40. The sequence of patterns 30–36 may be repeated after each pattern in the sequence is used. These sequences may also be specially designed to reduce the occurrence of artifacts from their repetitive temporal patterns.

### **SUMMARY**

10        Systems and methods of embodiments of the present invention comprise the creation and/or application of dither structures. These structures may be used to reduce the visibility of contouring and other artifacts in still and video images.

## BRIEF DESCRIPTION OF THE DRAWINGS

The following drawings depict only typical embodiments of the present invention and are not therefore to be considered to be limiting of its scope, the invention  
5 will be described and explained with additional specificity and detail through the use of the accompanying drawings in which:

FIG. 1 illustrates an image display system;

FIG. 2 illustrates another image display system;

10 FIG. 3 illustrates a three-dimensional dither structure;

FIG. 4 illustrates a multi-dimensional dither structure with multiple image characteristic channels;

FIG. 5 a multi-dimensional dither structure with multiple image characteristic channels and an Initial Reference Frame comprising multiple dither tiles;

15 FIG. 6 illustrates a general high-pass spatial and high-pass temporal power spectrum;

FIG. 7 illustrates the relationship between a sigma value and a dither value in some embodiments of the present invention;

FIG. 8 illustrates an exemplary spatial feedback function of some embodiments  
20 of the present invention;

FIG. 9 is a block diagram illustrating exemplary methods for creating a dither pattern tile set;

FIG. 10 illustrates a radial frequency spectrum of a dither array of some embodiments of the present invention;

FIG. 11 illustrates a temporal frequency spectrum of a dither array of some embodiments of the present invention;

FIG 12 illustrates a use of a dither pattern tile set wherein dither pattern tiles are arranged in a specific sequence; and

5        FIG 13 illustrates another use of a dither pattern tile set wherein tiles are put in a random spatial pattern, but used sequentially in the temporal dimension.

## DETAILED DESCRIPTION

Embodiments of the present invention may be used in conjunction with displays and, in some embodiments, in display algorithms that employ properties of the visual system in their optimization. Some embodiments of the present invention may comprise  
5 methods that attempt to prevent the contouring artifacts in displays that have too few gray levels. Some of these displays include LCD or similar displays with a digital bit-depth bottleneck. They may also be used with graphics controller cards with limited video RAM (VRAM). These bit-depth limitations can arise in the LCD display itself, or its internal hardware driver.

10 Some embodiments of the present invention include systems and methods comprising an anti-correlated spatio-temporal dither pattern, which exhibits high-pass characteristics in the spatial and temporal domains. Methods for creating these patterns comprise generation of a series of dither tiles for multiple image characteristic channels and the temporal domain.

15 In a non-limiting example, as shown in Figure 4, a different dither pattern tile 50, 52 & 54 may be generated for each of three RGB color channels and this set of three tiles 58 may be generated for a series of temporal frames 58, 60, 62 & 64. In this example, a multi-dimensional array of tiles is generated. In other embodiments, varying numbers of chrominance and luminance channels may be used and varying patterns may be used in  
20 successive frames in the temporal domain also.

In some embodiments of the present invention, as illustrated in Figure 5, a set of dither pattern tiles is generated one element at a time by successively designating each pixel value according to an anti-correlation or dispersion method, which may be referred to as a merit function. To initiate the procedure, an initial reference dither pattern or set

of initial reference dither patterns 70 may be used.

An initial reference dither pattern 72, 74 & 76 may be a dither tile with a random noise pattern, a pre-set pattern, a constant value across all pixels, a blank tile or some other fixed or random pattern. A set of initial reference dither patterns 72, 74 & 76 for multiple channels of an image, such as the R, G and B channels of an RGB image, forms an initial reference frame 70. Once the initial reference pattern or frame 70 is established, pixel values in the dither pattern tiles can be generated. To ensure that the generated pattern is high-pass, a dispersion-related merit function is used to place each pixel.

In this exemplary method, a first pixel 80 is placed in the red channel tile 78 of frame 1. According to the dispersion merit function, this pixel is placed at a point that is dispersed from the location of pixel values in the initial reference frame tiles 72, 74 & 76. This dispersion merit function can relate to values in same color channel or a combination of color channels. Each color channel tile in the initial reference frame may be weighted to give different channels priority over others.

Once the first pixel has been placed, other pixels can be placed according to the dispersion merit function. These subsequent pixels will be placed in a manner that is dispersed from the first pixel 80 and may also be dispersed from pixel values in the initial reference frame 70. Generally, pixel values in the actual dither pattern 78 being developed will have greater weight than those in the initial reference frame 70, however these weighting factors may vary for specific applications. Each dither pattern tile (i.e., 78) can be completed individually or a set of tiles making up a frame may be generated simultaneously. For example, a pixel may be placed in a red channel tile 78 followed by a pixel placement in a green channel tile 82 of the same frame followed by a pixel



placement in the blue channel tile 84 of the same frame. Alternatively, a single color channel tile may be completed before placement of pixel values in another color channel tile of the same frame.

In this manner, each frame's dither pattern tiles are generated with reference to the patterns already established in previous frames and/or the initial reference frame. As the process continues from frame to frame, the weighting of previous frames may vary. For example, the weight given to pixel values in the closest preceding frame may be higher than that given to the next closest preceding frame. In some embodiments the initial reference frame 70 may be used only to generate the first frame 86. In other embodiments the initial reference frame 70 may be referenced in the generation of multiple successive frames with or without weighting factors.

Typically, due to memory constraints, the number of dither pattern frames is much less than the number of frames in a video clip so a series of pattern frames is reused in sequence. This cycle makes the first frame of the sequence 86 immediately follow the last frame 90. Accordingly, if these frames are not correlated, visible artifacts may develop. To avoid this, the last frames in a sequence are generated with reference to the first frame or frames as well as the previous frame or frames. This helps ensure that the pattern is continuously high-pass throughout multiple cycles.

In an exemplary embodiment of the present invention a 32x32 spatial dither pattern tile is generated for each color channel for RGB application. This pattern is created for 32 temporal frames thereby yielding a 32x32x32x3 array. The size is not a factor in the overall function of some embodiments and many different dimensions may be used. A merit function is used to disperse the pixel values into a high-pass relationship. This high-pass relationship may exist spatially within a dither pattern tile,

spectrally across color channel tiles and temporally across successive frames. In order to achieve all these relationships, the location of a pattern pixel value must have feedback from other pixel values within the tile pattern, other color channel tiles within the frame and pixel values in adjacent frames. Dispersion or anti-correlation across color channels can help reduce fluctuation in luminance where human vision has the highest sensitivity.

Negative feedback is a way to control the pattern so that pixel values are equally spaced in space and/or time. As a non-limiting example, if a large dither value is assigned to a position A at (i, j, k), its neighbors will be forced to take smaller values because negative influence from the large value at A. The further away from A, the less the influence the value at A will have on another pixel designation.

Figure 6 is a diagram showing a mutual high-pass temporal and spatial relationship achieved in some embodiments of the present invention. In order to achieve a high-pass pattern similar to that shown in Figure 6 a variety of feedback functions and parameters may be used.

To define a dither pattern tile set several parameters must be defined. The spatial size of each tile (i.e.,  $M \times N$ ), the number of frames, L and the number of color channels must be designated. Each parameter has trade offs that must be balanced. However, embodiments of the present invention allow less resource intensive parameters to be used without perceptible degradation of the final image. The number of levels in the dither pattern set must also be determined. A level may correspond to a luminance value, such as a gray-scale value in a monochrome image, a value for the luminance channel in image formats with specific luminance channels (i.e., LAB, LUV) and other parameters related to the visual perception of a pixel. This number may vary significantly according to specific application factors. In some embodiments, the number of levels may be determined with reference to the number of input bits and the number of output bits. In

these embodiments, the number of levels may be determined by taking 2 to the power of the difference between the number of output bits and the number of input bits. In equation form this expression would be:

$$n = 2^{(b_{in} - b_{out})}$$

- 5           For example, for an LCD display with the capability to display 6 bits, but receiving an input signal with 10 bits of data, the number of levels would be

$$n = 2^{(10-6)} = 2^4 = 16 .$$

- When a display is linear, the dither values may be evenly distributed among each level. However, in many cases the display is not linear so the level distribution may be  
10   distributed in a non-linear manner. When the number of output bits is greater than 4 the non-linear effect is small so uniform distribution does not cause a large non-linear error. Accordingly, the number of pixel values may generally be distributed evenly among levels. However, for lower numbers of output bits and larger non-linearities (i.e., gamma > 2) more threshold values should be distributed in the lower portion of the threshold  
15   range to compensate for the non-linear gamma effect.

### Temporal Feedback

- Negative feedback is used to push the temporal frequency of the dither pattern into high frequencies. In some embodiments, for frame 1, since it is the first frame with no other  
20   frames to reference, the temporal feedback function, fMask, relates to an initial reference frame (IRF). The initial reference frame may comprise essentially any noise pattern. An IRF may comprise pseudo-random noise, alternating patterns, a field of constant pixel values, a blank tile or frame or any number of other “patterns.” In some embodiments, the IRF may be set to a uniform noise of amplitude 0.1.

For frame 2, frame 1 may be used as a feedback function. Frame 2 may also reference the IRF in some embodiments. For frame 3 and up, a temporal infinite impulse response (IIR) may be used in generating the feedback function, as shown in the following exemplary equation:

5 
$$fMask = fMask * IIRCoef + (1 - IIRCoef) * frame(T-1) \dots$$

The further away from the current frame, the less is the contribution to the feedback function.

For the last frame, since the dither pattern will repeat itself, in order to achieve a temporal high-pass relationship between the last frame and the first frame, the contribution  
10 of the first frame may be added to the temporal IIR filtering as:

$$fMask = fMask * IIRcoef + (1 - IIRcoef) * 0.5 * (frame(T-1) + frame(1))$$

While these particular embodiments have been found to perform well, many other methods may be used to disperse pixel values spatially and temporally.

## 15 Spatial Feedback

The idea behind spatial noise distribution is trying to evenly distribute the dither values so that there is minimum fluctuation in both luminance and chrominance when viewed from a certain distance. In some embodiments, the first dither value or pixel of the first level is entirely dependent on the fMask function and the initial reference tile or  
20 frame, when an IRF is used. In some embodiments, it will take the position of the maximum value in the IRF. In other embodiments, where a multiple channel IRF is used, cross-channel feedback from the IRF may cause this position to vary. Subsequent pixels are generally placed as far away as possible to all the previous pixels. This is equivalent to placing charged balls in a plane. Each ball is trying to repel other balls of the same

charge as far as possible. The new ball will end up in the least occupied space when all values are equal. The inverse distance-squared function may be used as a repellent function, which is equivalent to the repellent force between charges of the same type. The repellent function may be implemented with a convolution kernel as

5

$$k(x, y) = \frac{1}{\frac{x^2+y^2}{\sigma^2} + 0.5}$$

where  $x$  and  $y$  are the spatial coordinates, the constant 0.5 is used to prevent division by 0. It is also used to adjust cross color channel influence as described later. Sigma ( $\sigma$ ) defines the spatial extent of the repellent function. It may be level dependent. For the  
10 first level, we have more degrees of freedom to which to assign dither values, thus the sigma may take a larger value. At the midlevel, near half of the cells are assigned and sigma may take a smaller value. Figure 7 shows an exemplary relationship between sigma and the dither value level. This relationship works well in applications, however many other relationships including constant values may be used in embodiments of the  
15 present invention.

In some embodiments, the spatial feedback function may be referred to as the *sMask* function and may be expressed mathematically as

20

$$sMask(x, y, color) = img(x, y, color) ** k(x, y)$$

where  $**$  represents a convolution operation and  $img(x, y, color) = 1$  if a position is already assigned a dither value. To improve the speed, the convolution operation may be implemented in the frequency domain using Fourier transforms

$$sMask(x, y, color) = F^{-1} \{ F[img(x, y, color)] \bullet F[k(x, y)] \}$$

where  $F$  denotes a forward Fourier transform and  $F^{-1}$  denotes an inverse Fourier

- 5 transform. Whenever a new pixel is added, sMask may be recalculated to account for the presence of the new pixel value. Figure 8 shows a typical spatial feedback function that may be used in embodiments of the present invention. In Figure 8, the peaks 140 represent points where dither values have already been assigned.

#### 10 Cross Color Channel Feedback

- Since the luminance sensitivity of human vision is higher than chrominance sensitivity, it is important to optimize multiple color dither arrays so that the luminance fluctuation is minimized. As a non-limiting example, in an RGB image, for a given gray (luminance value), if the red dither value is assigned to a position, the green dither value
- 15 should also be repelled by the red dither value. Cross channel feedback can be implemented using a set of weighted spatial feedback functions, which may be implemented as follows:

$$\begin{bmatrix} cMask_r \\ cMask_g \\ cMask_b \end{bmatrix} = \begin{bmatrix} C_{rr} & C_{gr} & C_{br} \\ C_{rb} & C_{gg} & C_{bg} \\ C_{rb} & C_{gb} & C_{bb} \end{bmatrix} \times \begin{bmatrix} sMask_r \\ sMask_g \\ sMask_b \end{bmatrix}$$

20

where  $C_{ii}$  is the weight of one color feedback function to another color. Since the contribution to luminance is different for the three color channels, with green having the biggest contribution and blue the least, therefore, in some embodiments we can optimize

the weight so that  $C_{gg}$  is higher than  $C_{bb}$ . However, in many applications, this effect has been found to be small. Accordingly, in some embodiments, only two weights are implemented: off-diagonal weight  $C1$  and diagonal weight  $C2$ . At mid levels,  $C1$  is the smallest so that the cross channel feedback is very small. Various methods may be used to determine the best weighting values. Constant values may be used in some embodiments. These weights may also be determined using a level-dependent method. One embodiment of this is shown in the equations below.

$$C1 = ((\text{level} - \text{nLevels}/2)/\text{nLevels})^2 + 0.07$$

$$C2 = 1 - 2 * C1$$

#### Combination of Temporal and Spatial Feedback Functions

The temporal feedback function, spatial feedback function and cross-channel feedback function may be combined to form a merit function for determining the position of a dither pattern value. The location of the minimum or maximum of this merit function may be assigned a new dither value (level). When the level is small, most of the space is unassigned and it is easier to find the few positions that are already assigned. However, when the level number is close to the last level, most of the space is occupied and it is easier to find the holes that are not assigned. Thus the generation process may be divided into two steps:

For level  $\leq$  nLevels

$$\begin{aligned}
mask(x, y, color) &= 1 - fMask(x, y, color) + cMask(x, y, color) \\
find(x_0, y_0) \parallel mask(x_0, y_0, color) &= \min(mask(x, y, color)) \\
TA(x_0, y_0) &= level - 1 \\
img(x_0, y_0) &= 1
\end{aligned}$$

Some exemplary embodiments of the present invention may be explained with reference to Figure 9, which is a flow chart showing exemplary methods 100 for creation of a dither pattern tile set. In these embodiments a series of loop structures are used to perform repeated functions, however, alternative embodiments may use other recursive structures to implement these functions.

Initially, dither pattern tile set parameters 102 are designated to define the dimensions and characteristics of the tile set. Once the tile set is defined, each successive frame 104 is designated with reference to an initial reference frame and/or other image frames. In order to relate pixel values in a new dither pattern to other pixel values in preceding frames, an fMask function 106 is used. Depending on the position of the frame being designated, a different relationship or fMask function may be used as shown in the diagram 106, 108, 110 & 112.

In these particular embodiments, the first frame 106 will be designated with reference to an initial reference frame (IRF), which may be a random noise pattern or essentially any other pattern including a constant value tile or a blank tile. In some embodiments, the initial reference frame may simply be omitted and the first pixel value of the first frame may be placed by pseudo-random methods or other methods.

After the first frame of the dither pattern tile set has been established, the second frame may be established using an fMask function 108 that relates to the pixel values in the first frame. Subsequent frames may be established 110 with reference to one or more



of the preceding frames and the IRF. The fMask function for the last frame 112 references the pixel values in the preceding frames as well as the first frame, which will be used in a cycle immediately following the last frame.

Once the fMask function for a particular frame is determined, a dither pattern tile  
5 is initialized 114 and the process for establishing the first level 116 of values is commenced. When cross-channel feedback methods employ level-dependent weighting factors, these factors may be calculated for the particular level 118.

In these exemplary embodiments, a loop is entered to designate the number of pixels that have been allocated to that particular level 120. Another loop is entered to  
10 cycle through the color channels 122. These structures are merely exemplary for some embodiments of the present invention and may be modified in many ways for alternative embodiments.

For each pixel value in a particular level within a particular color channel tile, the feedback functions are aggregated to find the location of a dither pattern pixel value 124.  
15 This operation may comprise spatial feedback, cross color-channel feedback and temporal feedback as well as other factors. Once a pixel value has been designated, the feedback values are recalculated using the new pixel value as additional input 126. Subsequent pixel values will be repelled from that newly designated value as well. In these illustrative embodiments, the next color tile is then selected 128 and a pixel value is  
20 designated in that tile. This second color pixel value is determined 130 according to the merit function taking into account the location of the first pixel value in the first color channel. This pixel designation process is repeated until all pixel values for a particular level have been designated for each of the color channels.

When a level is fully designated for all color tiles, the next level is selected 132

and pixel values for that level are designated for all color channels. When all levels have been designated for all color channels the next frame is selected 134. The process then repeats for the next frame by calculating the appropriate fMask 112 temporal feedback function, cross-channel feedback values 118 and spatial feedback factors 126 as well as  
5 other calculations. Once all frames are designated, the entire dither pattern array is stored for use in video processing 136.

It should be noted that in alternative embodiments, not illustrated in Figure 9, dither pattern pixel values may be designated in other orders. As a non-limiting example, the pixel distribution loop 130 may reside within the color channel selection loop 128  
10 causing all pixels values for one level of a color channel to be designated before proceeding to the next color channel. As another non-limiting example, the level selection loop 132 may reside within the color selection loop 128. In effect, this alternative will cause a pixel value from each level to be placed in a color channel tile before proceeding to the next color channel. Many other variations in these processes  
15 may also be implemented by one skilled in the art based on the information described herein.

To determine the frequency characteristics of dither pattern arrays produced with embodiments of the present invention a Fourier analysis may be used. Figure 10 shows a graph of the radial frequency spectrum of one frame of an exemplary dither array. This  
20 demonstrates the spatial high-pass characteristics of a dither pattern. Figure 11 shows the temporal frequency spectrum of a dither array and demonstrates the temporal high-pass frequency characteristics of the array.

Some embodiments of the present invention may also employ a tile stepping method as illustrated in Figure 12 for further reduction of the possibility of visible

artifacts. In these embodiments, a spatio-temporal array of dither pattern tiles 150 may be used. These dither pattern tiles 150 are typically smaller than the image to which they are applied in order to reduce memory size. The smaller tiles can cover the image in a tile pattern that uses the same tiles repeatedly. In some applications, the same tile is used repeatedly across the image as shown in Figure 3. However, this method can result in visible artifacts caused by the repeated pattern. This problem may be reduced or eliminated by using tiles from multiple successive frames. This method can be employed in the spatial and temporal dimensions. As shown in Figure 12, tiles can be incremented spatially across an image 152 starting with a first tile frame 160 and then using each successive tile frame 161, 162 & 164 to fill out the tile pattern across the image 152. This pattern of successive tile frames can be employed in the temporal direction as well. In the next successive image frame 154, the tile frame succeeding the tile frame used in the prior image frame at any given tile location is used. For example, when a first tile frame 160 is used in the top left position in a first image frame 152, the next successive tile frame 161 is used at that location in the next image frame 154. Similarly, the second tile position in the first frame 152 is occupied by the second tile frame 161 and that position in the second image frame 154 is occupied by the third tile frame 162. The same pattern is repeated for each tile position and each image frame. Once the number of tile frames is exhausted, the tile set order is repeated.

In other embodiments of the present invention, the tile pattern in a particular frame may be varied beyond a sequential spatial order across the rows. In some embodiments, the tiles may be dispersed in a random spatial order across a frame. Once this random spatial pattern is established in the first frame, the tiles in the next temporal frame and subsequent frames will follow a sequential temporal order such that the tile

corresponding to the position of a tile in the first frame will be the next sequential tile in the temporal order established in the dither tile structure. These embodiments are illustrated in Figure 13 where a dither tile set 170 is established with tile frames 0 through 3 (172-178) shown in sequential temporal order. Tile set 170 will typically

5      comprise many other frames as well, but the quantity illustrated is limited to 4 for simplicity of explanation. In a first image frame 180, tiles 172-178 and other tiles in a set are dispersed randomly across the frame 180. In the next image frame, p+1 (182), the tile used at any particular location is the next tile in temporal order from the tile used at that location in the previous frame. For example, at the top left tile location 184 in frame “p”

10      180, dither tile 6 is used as randomly placed. For the tile at that location 194 in frame “p+1” 182, the next tile in temporal order established in the dither tile structure 170, frame 7, is used. Likewise, for the second tile in the first row 186 of frame “p” 180, tile 2 is used and the next tile, tile 3 is used for that location 196 in frame “p+1” 182. Of course, other non-random and pseudo-random patterns may be employed as well.

15              Some embodiments of the present invention may make use of the oblique effect of the human visual system. The contrast sensitivity function of the human visual system is dependent on the viewing orientation. Vertical and horizontal sensitivity are higher than diagonal angles such as 45 degrees. To take advantage of this effect, the dither pattern may be designed to have its power spectra peak at 45 degrees. The convolution

20      kernel of embodiments of the present invention can take advantage of this property. Instead of using Euclidian distance, we can use city block distance in the repellent function as shown in the equation below:

$$k(x, y) = \frac{1}{\frac{(|x|+|y|)^2}{\sigma^2} + 0.5}$$